Fine-Grained Complexity of Temporal Problems

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Temporal Problems

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Question. Is there an assignment $f : V \to A$ that satisfies every $R(v_1, \ldots, v_t) \in C$?

Point Algebra

CSP Example 1: Point Algebra

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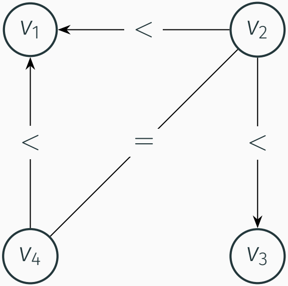

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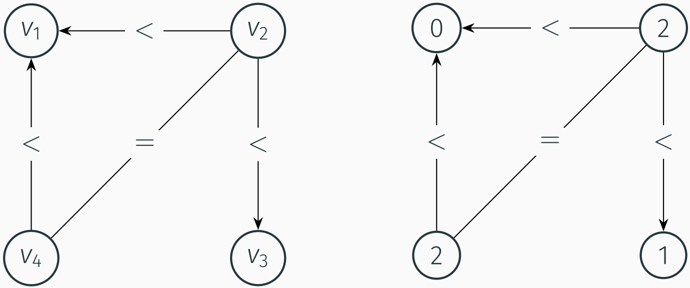

Simple Temporal Problem

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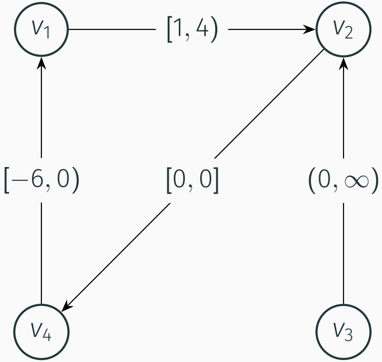

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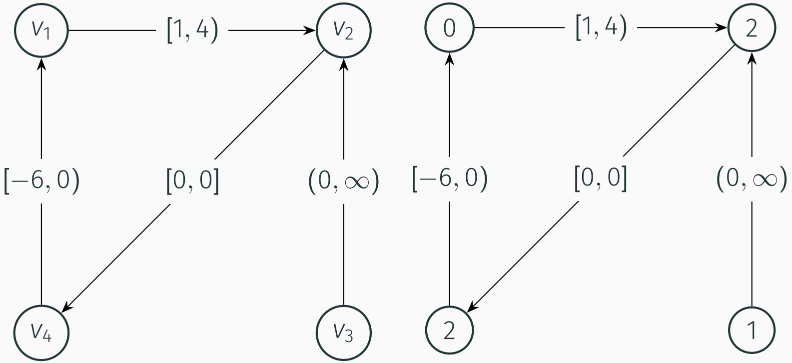

Disjunctive Temporal Problems

CSP Example 3: Disjunctive Temporal Problem

D_ω contains $R(v_1, \ldots, v_t) \equiv \bigvee_{\ell=1}^{m} v_{i_\ell} - v_{j_\ell} \in I_\ell$ for $t, m \geq 1$, $i_\ell, j_\ell \in \{1, \ldots, t\}$ and intervals with endpoints $I^-, I^+ \in \mathbb{Z} \cup \{\pm\infty\}$.

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Allen's Interval Algebra with Unit Intervals

Classical Complexity of Temporal Problems

CSP(S) (Simple Temporal Problem) is in P.

CSP(D_ω) (Disjunctive Temporal Problem) is NP-hard.

CSP(D_2) (Binary Disjunctive Temporal Problem) is NP-hard.

CSP($\mathsf{D}_{\omega,k}$) is NP-hard for all k.

CSP($\mathsf{D}_{2,k}$) is in P for $k = 0$ and NP-hard for $k \geq 1$.

Fine-Grained Complexity

Classical Complexity

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There may be a sequence $c_1 < c_2 < \dots$ such that CSP($\mathsf{D}_{2,k}$) is solvable in $O(c_k^n)$ time.

Selected Proofs

Upper bounds on D_2: Part I

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Analysis: $nk^n = 2^{n(\log n + \log k)}$ guesses, $O(n^2)$ time per guess. $\square$

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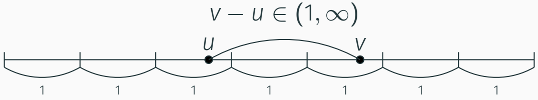

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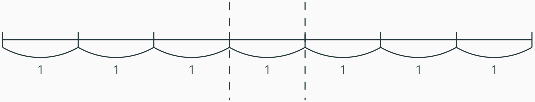

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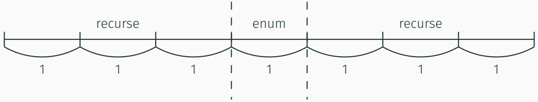

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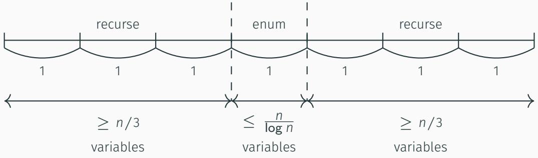

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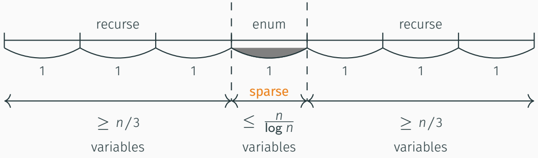

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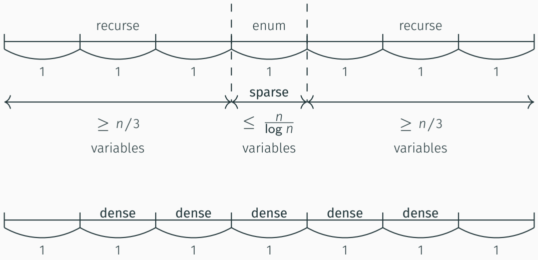

Theorem 2: $\mathrm{CSP}(\mathsf{D}_{2,k})$ is solvable in $2^{O(n \log \log n)}$ time for any fixed k.

Proof idea for $\mathrm{CSP}(\mathsf{D}_{2,1})$.

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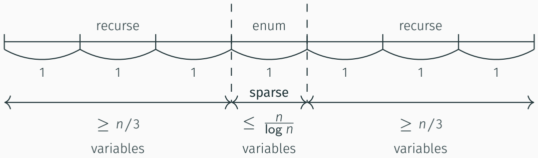

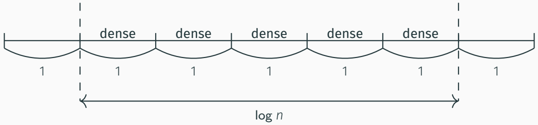

Upper bounds on D_2: Part II

Theorem 2: CSP($D_{2,k}$) is solvable in $2^{O(n \log\log n)}$ time for any fixed k.

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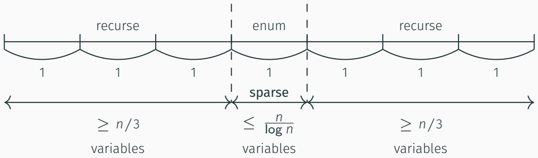

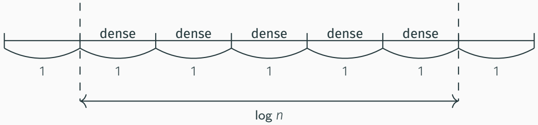

Dominant factor: $O(\log n)^n = 2^{O(n \log\log n)}$ time.

Conclusion

Allen's Interval Algebra

Prominent formalism for qualitative temporal reasoning.

Allen's Interval Algebra

Prominent formalism for qualitative temporal reasoning.

Basic relation		Example	Endpoints
I precedes *J*	p	*iii*	$I^+ < J^-$
J preceded by *I*	p^{-1}	*jjj*	
I meets *J*	m	*iiii*	$I^+ = J^-$
J met-by *I*	m^{-1}	*jjjj*	
I overlaps *J*	o	*iiii*	$I^- < J^- < I^+$,
J overl.-by *I*	o^{-1}	*jjjj*	$I^+ < J^+$
I during *J*	d	*iii*	$I^- > J^-$,
J includes *I*	d^{-1}	*jjjjjjj*	$I^+ < J^+$
I starts *J*	s	*iii*	$I^- = J^-$,
J started by *I*	s^{-1}	*jjjjjjj*	$I^+ < J^+$
I finishes *J*	f	*iii*	$I^+ = J^+$,
J finished by *I*	f$^-$1	*jjjjjjj*	$I^- > J^-$
I equals *J*	e	*iiii*	$I^- = J^-$,
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I during J	d	iii	$I^- > J^-$,
J includes I	d^{-1}	jjjjjjj	$I^+ < J^+$
I starts J	s	iii	$I^- = J^-$,
J started by I	s^{-1}	jjjjjjj	$I^+ < J^+$
I finishes J	f	iii	$I^+ = J^+$,
J finished by I	f^-1	jjjjjjj	$I^- > J^-$
I equals J	e	iiii	$I^- = J^-$,
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Constraints may involve any disjunction of basic relations.

Unit Interval Algebra

Intervals have unit length.

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Equivalent to CSP($\mathsf{D}_{2,1}$):

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Questions

Corollary: Unit Interval Algebra is solvable in $2^{O(n \log \log n)}$ time.

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Question: Is Unit Interval Algebra solvable in $2^{O(n)}$ time?

Question: Is Allen's Algebra solvable in $2^{O(n)}$?

Thank you!